THESIS PROPOSAL

Title: Global and Periodic solution in time and numerical analysis for bidomain and tridomain Models in Electrocardiology.

CNU section: Section 26, mathématiques appliquées et applications des mathématiques.

Affiliation institutions: École Centrale de Nantes (ECN) and École supérieure d'ingénieurs Léonard de Vinci (ESILV).

Doctoral schools affiliation: École doctorale Mathématiques et Sciences et Technologies de l'Information et de la Communication (MaSTIC) **In codirection** with, école doctorale SMI, ENSAM.

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1. Introduction

During the last twenty years, the modelling of the electrophysiological (or electrophenomenological) activity of the heart has seen a sustained interest and research activity on the part of researchers in biophysics and biomathematics. More specifically, the derivation of mathematical models of the type of partial differential equation systems describing the interaction between the propagation of electrical potentials coupled to physiological ionic patterns and the mechanical deformation of the heart tissue. This coupling is supposed to describe the electrophysiological waves in the heart tissue. One class of models has been intensively studied, the Bidomain models. Rigorous derivations from microscopic models have been obtained, as well as mathematical results concerning the existence and uniqueness of solutions in local or global time for phenomenological and physiological ionic models.

Here is an indicative bibliography, which together with the bibliography cited in these articles, constitutes a database on the mathematical state of the art concerning these models: [6] [5], [4], [3].

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Key words and phrases. Bidomain and tridomain models, reaction–diffusion system, global existence and periodic solution, mixed finite element and volume schemes, stabilization and control, numerical simulation.

2. Questions considered in this thesis

In this thesis we propose to contribute to the advancement of the mathematical and numerical analysis of these models as well as to propose models with controls that we will justify mathematically. More specifically, the topics that will be addressed, also adjusted, modified or completed, depending on the progress and the results obtained, are :

(1) Part 1: Some extensions of existing theoretical results.
   (a) • Extension of the existence and uniqueness results for three-domain models, obtained in [2] and [1] by homogenization, from solutions at the microscopic level in the case of phenomenological laws. We propose to make a direct study of the existence problem on macroscopic models with physiological and not only phenomenological ionic laws. An important mathematical difficulty intervenes in this extension linked to the loss of the Lipschitzian character of the functional, intervening in the ionic law, linking the various unknowns intervening in the PDE's system. The idea is either to refine the compactness lemmas used in [2] or to define approximation models, based on the approximation of the ionic law, to construct a sequence of solutions converging to a solution of the initial problem.
   (b) • Construction of periodic solutions, for bi-domain and tri-domain models. At this level, the field remains quite virgin and many results can be obtained by adapting methods used for reaction-diffusion type PDE systems. We can start by looking at the rare articles concerning periodic solutions for systems in the same theme, such as : [7], [10], [8],[9].

(2) Part 2: Numerical analysis and simulation.
   A hybrid numerical scheme using finite elements and finite volumes for models in simple situations will be proposed.
   (a) • We propose firstly the construction of a Two-Point Flux Approximation (TPFA) finite volume method to approximate the solution of the monodomain model. The aim here is to capture the periodic solution on the real geometry of the heart.
   (b) Secondly, we propose a combined finite volume/finite volume method to approximation the periodic solution for the bidomain model.

(3) Part 3: Stabilization and control problem for the bidomain model.

References

[1] Fakhrielddine Bader, Mostafa Bendahmane, Mazen Saad, *Microscopic tridomain model of electrical activity in the heart with dynamical gap junctions. Part 2- Derivation of the macroscopic tridomain model by unfolding homogenization method, Asymptotic analysis* (2022), 1-32.

[2] Fakhrielddine Bader, Mostafa Bendahmane, Mazen Saad, *Microscopic tridomain model of electrical activity in the heart with dynamical gap junctions. Part 1- Modeling and Well-posedness, Acta Appl. Math.* 179 (2022), Paper No. 11, 37 pp.

[3] Fakhrielddine Bader, Mostafa Bendahmane, Mazen Saad, *Three scale unfolding homogenization method applied to cardiac bidomain model, Acta Appl. Math.* 176 (2021), Paper No. 14, 37 pp.

[4] Fakhrielddine Bader, Mostafa Bendahmane, Mazen Saad, *Derivation of a new macroscopic bidomain model including three scales for the electrical activity of cardiac tissue, Journal of Engineering Mathematics volume 131, Article number: 3 (2021)*

[5] M. Bendahman, Fatima Mroue,M. Saad, R. Talhouk *Unfolding homogenization method applied to physiological and phenomenological bidomain models in electrocardiology, Nonlinear Analysis: Real World Applications 50 (2019) 413-447.*
[6] M. Bendahman, Fatima Mroue,M. Saad, R. Talhouk *Mathematical analysis of cardiac electromechanics with physiological ionic model, DCDS, Series B, Volume 24, Number 9, September 2019.*
[7] Y. Giga, N. Kajiwara, K. Kress, *Strong time-periodic solutions to the bidomain equations with arbitrary large forces, Nonlinear Analysis: Real World Applications, Volume 47, June 2019, Pages 398-413.*
[8] A. Fraguela, R. Felipe-Sosa, J. Henry, M. F. Marquez *Existence of a T -Periodic Solution for the Monodomain Model Corresponding to an Isolated Ventricle Due to Ionic-Diffusive Relations, Acta Applicandae Mathematicae volume 177, Article number: 4 (2022).*
[9] H. Matano, Y. Mori, M. Nara, K. Sakakibara, *Asymptotic Behavior of Fronts and Pulses of the Bidomain Model, SIAM Journal on Applied Dynamical Systems, Vol. 21, Iss. 1 (2022)*
[10] M. Hieber, N. Kajiwara, K. Kress, P. Tolksdorf, *The periodic version of the Da Prato–Grisvard theorem and applications to the bidomain equations with FitzHugh–Nagumo transport, Annali di Matematica Pura ed Applicata (1923 -) volume 199, pages 2435–2457 (2020)*

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