DYNAMICS OF THE SCHRÖDINGER EQUATION IN SUB-RIEMMANIAN GEOMETRY

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The proposed PhD thesis consists in studying the dynamics of the Schrödinger equation in the case where the Laplacian is associated with a sub-Riemannian structure using semiclassical methods [Zwo12]. As a starting point, the question of observability of the Schrödinger equation in dimension 3 will be addressed in the case of a general sub-Riemannian contact structure building on the recent works [CdVHT18, BS22, FKF21, AS23]. The relation with the dynamics of the semiclassical magnetic Schrödinger equation will also be considered in light of the recent developments [AR23, Riv23].

Required background.

- Spectral theory;
- Fourier analysis (ideally microlocal/semiclassical analysis);
- Hamiltonian dynamical systems;
- Differential geometry.

References

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[CdVHT18] Y. Colin de Verdière, L. Hillairet, and E. Trélat. Spectral asymptotics for sub-Riemannian Laplacians, I: Quantum ergodicity and quantum limits in the 3-dimensional contact case. *Duke Math. J.*, 167(1):109–174, 2018.

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