PhD projet

March 11, 2024

Title: Khovanov homology and non-compact exotic surfaces

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Description du projet de thèse

The topology of 4-manifolds is quite weird. 4 is the only dimension where a topological manifold can support infinitely many pairwise non-diffeomorphic differentiable structures. It is also the only dimension where $\mathbb{R}^n$ can support more than one smooth structure. (In fact, $\mathbb{R}^4$ supports *uncountably* many pairwise non-diffeomorphic smooth structures.)

Surfaces in 4-manifolds, either embedded, immersed, or with conical singularities, can be used to detect these kinds of exotic phenomena, but they are also an interesting subject of their own. For instance, Gompf recently showed that there are exotic embeddings of $\mathbb{R}^2$ into $\mathbb{R}^4$ [Gom23].

The goal of this thesis project is to use Khovanov homology to detect this type of phenomena. Khovanov homology is a combinatorial tool to study knots in the 3-sphere and cobordisms between them. It has been successfully used to study (properly) embedded compact surfaces [Ras10, HS22].

The strategy consists in adapting some ideas developped by Gadgil [Gad10]. Gadgil worked in the context of open 4-manifolds using Heegaard Floer homology. These ideas can be exploited to study non-compact surfaces in $\mathbb{R}^4$ using Khovanov homology.

References

[Gad10] Siddhartha Gadgil. Open manifolds, Ozsváth-Szabó invariants and exotic $\mathbb{R}^4$'s. *Expo. Math.*, 28(3):254–261, 2010.

[Gom23] Robert E. Gompf. Topologically trivial proper 2-knots. to appear in Geom. Topol., 2023.

[HS22] Kyle Hayden and Isaac Sundberg. Khovanov homology and exotic surfaces in the 4-ball. arXiv preprint arXiv:2108.04810, 2022.

[Ras10] Jacob Rasmussen. Khovanov homology and the slice genus. *Invent. Math.*, 182(2):419–447, 2010.