PhD project

Universality of fluctuations for interacting particle systems

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The main objective of this project is to study universal behaviors in physical systems with underlying microscopic stochastic dynamics, based on the Edward–Wilkinson (EW) and Kardar–Parisi–Zhang (KPZ) universality classes. We will mainly consider a class of interacting particle systems, the exclusion processes. They describe particles evolving on a lattice through nearest-neighbor interactions satisfying an exclusion rule at each site.
The concept of universality emerged in statistical mechanics in the study of critical phenomena. The singular behavior of a system near a critical point is characterized by critical exponents. These exponents describe the non-analyticity of various thermodynamic observables. It can be seen empirically that different systems can be described by the same set of critical exponents. These systems are said to belong to the same universality class. In recent decades decades, the concept of universality class has been widely studied and exploited: in particular, the grouping of non-equilibrium systems. In this context, the universality classes EW and KPZ play a central role. Consider a stochastic interface model, described by a function $h(x,t)$, which at each instant t gives the height profile of the interface. We can characterize the universality class to which a process belongs by looking at the space-time renormalization with dynamic exponents α and z, i.e. $h^{\epsilon}(x,t) := \epsilon^{-1}h(\epsilon^{-1/\alpha}x, \epsilon^{z/\alpha}t)$. It is a conjecture that models sharing the same dynamic behavior should converge to the same universal process $H(x,t) := \lim_{\epsilon\to 0} h^{\epsilon}(x,t)$ under the rescaling operation $1 : 1/\alpha : z/\alpha$.

Physical and mathematical systems that are described by Gaussian statistics, due to linear growth and the absence of spatial correlation, belong to the universality class EW, whose scale invariance is 1:2:4. The KPZ universality class should include models evolving according to random dynamics characterized by slope-dependent growth, a smoothing mechanism and short-range interactions. The KPZ scaling is 1:2:3.

We plan to study density field fluctuations for particle systems with one or more species (corresponding to one or more conserved quantities — e.g. the mass densities for each species) with the aim of showing that the scaling limits of density field fluctuations are described by statistics of the KPZ class and beyond.

In fact, the statistical picture describing the fluctuations of exclusion processes and random interfaces cannot be illustrated by the EW and KPZ classes alone. In the case of multi-component systems, the scenario is much broader and less well understood; this problem has arisen, in particular, in the context of nonlinear oscillator chains [Spohn '14]. The theory of nonlinear fluctuating hydrodynamics has provided a complex phase diagram of universality classes for multi-component systems. Depending on the details of the models, the fluctuations of the conserved fields can be in the EW, KPZ class, or in a discrete family of Lévy universality classes whose dynamic exponents are given by the quotient of consecutive Fibonacci numbers $z = 3/2, 5/3, 8/5, \ldots$.

The aim is to understand the phenomenon of crossover between universality classes for multi-component systems. So far, rigorous results have been proved only for $z = 3/2, 5/3$. Our intention is to extend the derivation of these universal laws to other universality classes (i.e. other dynamical exponents), by defining microscopic processes whose behavior for large times, encoded in universal SPDEs, shows a transition between universality classes.