Controllability of the equation governing elastohydrodynamics of a flexible magnetic micro-robot

Research context. This PhD project in applied mathematics tackles issues in control theory, particularly controllability and stabilization, both in finite and infinite dimension as well as the link between them. The dynamical systems taken into consideration are modelling the dynamics of elastic filaments in a viscous fluid, with a lookout towards applications to medical micro-robotics and bio-inspired locomotion.

The research on fabrication and promising medical usage of microscopic robots has been rapidly developing in the last decades [7], calling for advanced models for the complex fluid-structure interaction problems involved and precise studies on their guidance. In particular, the guidance by an external magnetic field of flexible robots inspired by biological flagella constitutes an important challenge.

When adding a term accounting for a prescribed magnetic field (or, for that purpose, any other form of control over the filament dynamics) to these systems, we can see them as control systems, which can in turn be studied from the point of view of mathematical control theory. In particular, one can ask whether the system is *controllable* (the formal existence of controls which allow to steer the system to any target around some equilibrium state) or *stabilisable* (the capacity to affect the stability properties of equilibria), with both these properties being crucial to inform the ability of a system to be effective in various applications.

In order to model the elastohydrodynamics of filaments at microscopic scale, one can choose amongst two categories of models: a continuum formulation for the elastic material, yielding a nonlinear partial differential equation (PDE), or a discrete formulation based on a mechanical description of the filament by rigid links, with the resulting system of governing equations being an ordinary differential equation (ODE); see Figure 1.

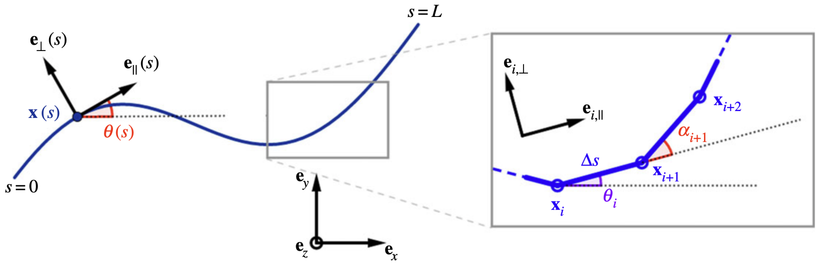


Figure 1: The continuous and discrete versions of an elastic filament.

Consequently, the equivalence (or the absence thereof) of controllability and stabilisation properties for the ODE and PDE elastohydrodynamic models constitutes an important question within the field, and its generalisation to broader classes of control systems stands as a leading research direction for control theory.

In turn, the main goal of this project is to establish formal equivalence results between control properties for the finite- and infinite-dimensional models. This long-term goal is underlain with several questions concerning the ODE and PDE models separately, defining intermediate milestones.

Current state of the field. There is a large number of studies on microswimmer control in the literature, among which many focus on slender swimmers inspired by the pioneer Purcell model [5]. However, most of them consider shape control, which is relatively unrealistic from a robotic point of view. Consistent theoretical results on controllability of more realistic models make for an important challenge for the field. In particular, the magnetic control case is, by far, less well understood.

Regarding the finite-dimensional model, controllability results have been obtained in the last few years [4], but a full investigation of non-trivial equilibria depending on the number of links and magnetisations is still lacking. Moreover, the continuum model with magnetism constitutes uncharted territory from the point of view of control theory, with promising discoveries to be made on its properties and behaviour using a combination of PDE analysis, techniques of nonlinear control and numerical simualtion, as explained below.

The idea of linking the controllability and stabilisability properties of infinite-dimensional models and a finite-dimensional version (typically, a spatial discretisation) was first investigated in [3, 2]. Indeed, the method proposed in these articles consists in adapting the classical numerical schemes (finite-element, finite differences, etc...) so that control properties, such that the observability or the controllability, are preserved.

Objectives and methods

Finite-dimensional model: "N-link swimmer"

- Modelling and study of the dynamical system and its equilibria, with respect to the magnetisations: stability, bifurcations.
- Controllability (Sussmann condition, etc.)
- (Long-term) Optimisation of the magnetic distribution for various objectives; formalisation of the problem and choice of an adapted optimisation framework.
- (Long-term) Proposal for an extended 3D version, possibly based on Cosserat rods.

Infinite-dimensional model: "Magneto-elasto-hydrodynamics"

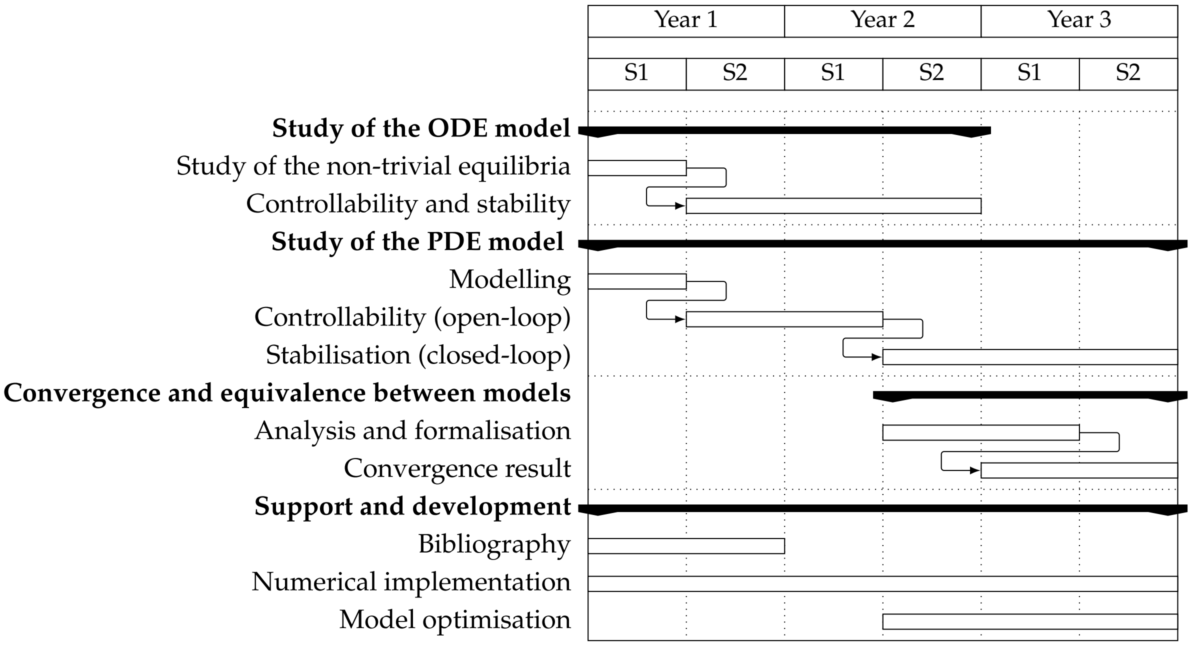
- Formal derivation of the model with a magnetic term.
- Bibliographical research to classify and rationalise modelling assumptions for hydrodynamics, elasticity and magnetics.
- Controllability of the open-loop system at equilibrium with the return method [1].
- Study of the closed-loop system, linearisation at equilibrium.
- Design of a numerical code to support the theoretical analysis.

Bridge ODE and PDE control

- Define notions of equivalence between the finite- and infinite-dimensional models, relying on recent works by Mori and Ohm [6] and Levillain et al. (in preparation)
- Establish under which conditions the PDE model can be replaced with its discretised version for controllability purposes.

Expected results The results, broken down into theoretical contributions on control systems and explicit results on the micro-organism/micro-robot system, are expected to appeal to communities of applied mathematics, automatic control and micro-locomotion physics. Several high-impact papers or conference proceedings adjusted for each field can be envisioned.

Provisional program



Working Context. The PhD will be co-advised by Franck Plestan (Professor, LS2N), Swann Marx (CR CNRS, LS2N) and Clément Moreau (CR CNRS, LS2N). The doctoral student will be hosted by the CODEX team in the LS2N laboratory, located in École Centrale Nantes. Starting date: September 2024.

Required Skills. Candidates should hold a MSc. degree in applied mathematics or equivalent, with a solid knowledge in partial differential equations and mathematical modelling. Basic programming skills are required. A more developed experience in numerical simulation will be appreciated.

In addition, knowledge in one or several of the following fields is highly appreciated:

- Control theory in finite and infinite dimension
- Fluid mechanics, condensed matter physics

Knowledge of French is not required.

References

[1] Coron, J. M. (2007). Control and nonlinearity (No. 136). American Mathematical Soc.

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[7] Zhou, H., Mayorga-Martinez, C. C., Pané, S., Zhang, L., & Pumera, M. (2021). Magnetically driven micro and nanorobots. Chemical Reviews, 121(8), 4999-5041.